

# Parts and Wholes

Adapted from a presentation by

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# Mereology

- Ground mereology: parthood is a reflexive partial order

$x$  is a part of  $x$ .

If  $x$  is a part of  $y$  and  $y$  is a part of  $x$  then  $x=y$ .

If  $x$  is a part of  $y$  and  $y$  is a part of  $z$  then  $x$  is a part of  $z$ .

# Mereology

- $x$  is proper part of  $y$ :  $x$  is part of  $y$  and  $y$  is not part of  $x$
- $x$  overlaps  $y$ : there is a part of  $x$  that is also a part of  $y$
- $x$  and  $y$  are disjoint:  $x$  and  $y$  do not overlap

# Mereology

- Binary Product:  $x \cdot y$ : individual that is part of both  $x$  and  $y$  and any common part of  $x$  and  $y$  is a part of the product.
- Binary Sum:  $x + y$ : individual that overlaps something iff it overlaps at least  $x$  or  $y$ .
- Difference:  $x - y$ : largest individual contained in  $x$  that has no part in common with  $y$ .

# Mereology

- Generalized Product:

$\prod x 'Fx'$  : product of all objects satisfying F

- Generalized Sum:

$\sigma x 'Fx'$  : sum of all objects satisfying F

# Mereology

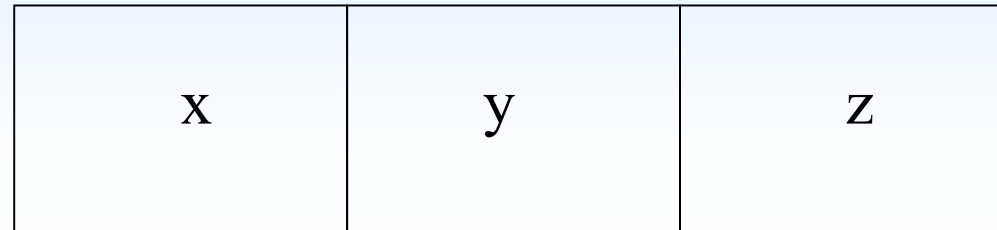
- Universe  $U$ : the sum of all objects
- Complement: the complement of  $x$  is  $U-x$ .

# Example

$$u = x + y$$

$$v = x + z$$

$$w = y + z$$



x is proper part of u  
x is proper part of v

u overlaps v

x and y disjoint  
y and z disjoint  
x and z disjoint

$$u \cdot v = x$$
$$x + w = u + v$$
$$u - w = x$$

# Classical Extensional Mereology

Axioms:

- 0: any axiom set sufficient for first-order predicate calculus with identity
- 1: if  $x$  is proper part of  $y$  then  $y$  is not a proper part of  $x$
- 2: proper part is transitive
- 3: if  $x$  is a proper part of  $y$  then there is a proper part  $z$  of  $y$  that is disjoint from  $x$  (weak supplementation principle)
- 4: if there are objects satisfying  $F$  then there is an  $x$  such that for all  $y$  it holds that  $x$  overlaps  $y$  iff there is a  $z$  satisfying  $F$  that overlaps  $y$  (existence of sums).

# Classical Extensional Mereology

Ground mereology

+ weak supplementation principle

+ existence of sums

strong supplementation: if  $x$  is not a part of  $y$   
then there is a part  $z$  of  $x$  that is disjoint  
from  $y$ .

# Classical Extensional Mereology

## Theorems

- $x$  is not a proper part of  $x$ .
- $x = y$  iff  $x$  is part of  $y$  and  $y$  is part of  $x$
- Transitivity of part-of
- If  $x$  is proper part of  $y$  and  $y$  is part of  $z$ , then  $x$  is proper part of  $z$

# Classical Extensional Mereology

## Theorems

- $x = y$  iff  
for all  $z$ :  $z$  is part of  $x$  iff  $z$  is part of  $y$
- $x = y$  iff  
for all  $z$ :  $x$  is part of  $z$  iff  $y$  is part of  $z$

# Classical Extensional Mereology

Atoms:  $x$  is an atom if it has no proper parts

Possible axioms:

1. Every individual has an atom as part.
2. Every individual has proper parts.
3. hybrid of 1 and 2.

Classical extensional mereology is neutral.

# Extensional Mereology - criticisms

- There are senses of part-of that are not transitive.
- There is no guarantee of the existence of sum-individuals.
- The criterion saying that individuals having all parts in common are identical, is generally false.

# Extensional Mereology - criticisms

- There are senses of part-of that are not transitive.

*The basic broad part-of is transitive. The non-transitivity comes from narrowing or specifying the part-of relation with things such as function.*

# Extensional Mereology - criticisms

- There is no guarantee of the existence of sum-individuals.

*Existence of sums does not make theory inconsistent.*

Problem of applying principle to real world.

May work well for portions and regions.

# Extensional Mereology - criticisms

- The criterion saying that individuals having all parts in common are identical, is generally false.

Example:

Tibbles  $\langle \rangle$  Tib + Tail

Tibbles and Tib + Tail share all parts

# 'Minimal' Requirements

(Simons 1987)

- Asymmetry of proper part
- Transitivity of proper part
- Weak supplementation principle

# Approaches from linguistics

- Winston, Chaffin, Hermann
- Iris, Litowitz, Evens
- Gerstl, Pribbenow

# Winston – Chaffin - Hermann

- Based on properties
  - Functional
  - Homeomericous
  - Separable

# Winston – Chaffin - Hermann

- Integral object – component
- Collection – member
- Mass – portion
- Stuff – object
- Activity – feature
- Area – place

# Integral object – component

- Functional
- Not homeomericous
- Separable
  
- *Cup – handle*
- *Car - wheel*

# Collection – member

- Not functional
- Not homeomericous
- Separable
  
- *Forest – tree*
- *Deck - card*

# Mass – portion

- Not Functional
- Homeomericous
- Separable
  
- *Pie – slice of pie*
- *Salt – grain of salt*

# Stuff - object

- Not functional
- Not homeomericous
- Not separable
  
- *Steel - bike*

# Feature - activity

- Functional
- Not homeomericous
- Not separable
- *Paying - shopping*

# Area - place

- Not functional
- Homeomericous
- Not separable
  
- *Sweden – Linköping*
- *Desert - oasis*

# Relation to other semantic relations

